ABSTRACT - Reliability analysis has been supporting energy operations planning studies in a very significant way, specially to obtain predictive indexes of peak load supply. During the last couple of years, due to high growth rates of peak load, Brazilian organism for Coordination of the Interconnected Operation (GCOI) decided to implement reliability studies in the short-term operations planning. The multi-area methodology that has been used in the long-term planning was extended and now, important information to the short-term planning, like probability distribution function of the EPNS, is available.

Keywords - Reliability analysis; Monte-Carlo simulation; Linear Network flows; Multi-area model; Performance indexes.

INTRODUCTION

Since 1989, generation reliability studies have been supporting the operations planning process in the Brazilian electric systems with two major roles: as a probabilistic criterion to obtain power interchange contracts between utilities and as reliability analysis of the generation system to evaluate performance indexes like LOLP, EPNS, LOLF, LOD of the peak load supply.

Until 1996, these studies were performed only in the long-term operations planning. Mid and short-term planning steps were not concerned with these aspects, mainly because of a structural surplus of peak capacity in the Brazilian system.

In the past few years, the delay and lack of investments postponed the construction of new power plants, and now utilities are facing difficulties to supply peak load on daily operation scheduling. In addition to these facts, annual load growth rates are around 6%, and in real time operation, utilities are facing load forecasts errors up to 10%.

This scenario urged the necessity of improving and implementing tools for short-term reliability analysis to gather new information to the planning process, like probabilistic indexes, consideration of interconnection constraints, hydroplants head variations and load forecasts deviations.

On the beginning of 1996, GCOI decided to introduce generation reliability studies on short-term operations activities in order to evaluate the peak load supply capacity of the interconnected system in a different way.

This process has been divided in two steps: (i) the use of a single bar methodology, because of its simplicity and also as an introduction of the subject and new concepts to the personnel involved in the elaboration of the planning studies (the Working Group for Operation Scheduling - GTPR); the model uses a mixed Monte-Carlo / analytical approach to solve the problem; (ii) migration to a multi-area modeling, based on a Monte-Carlo simulation approach that considers the most important interconnection constraints.

This paper presents a brief discussion of the present methodology and the results obtained in the long-term studies in 1996 and the short-term planning studies during 1997.

MULTI-AREA REPRESENTATION OF A POWER SYSTEM

A multi-area system can be represented by a linear network flow, where the nodes represent each area, and the arcs represent the interconnections among them. The generation capacity in each area is modeled as an arc coming into the area node from a fictitious "source" node S. Similarly, the area load is represented as an arc leaving the respective node to a "terminal" node, T. Figure 1 illustrates a system composed of two areas, with the capacity arcs S-1, S-2, the load arcs 1-T, 2-T and the interconnection arc 1-2.
The capacities associated to each arc are random variables, and can be obtained from the combination of the individual states of the equipment (generators and interconnections) and load levels. For example, the capacity of each generation arc is given by the sum of the available capacity of the generating units in the corresponding area. Its probability and incremental frequency distribution, also known as capacity outage probability and frequency table - COPFT, is calculated from the convolution of the capacity distributions of the individual generators [1,2].

A system state can be represented by a vector \( x = (x_1, x_2, \ldots, x_m) \), where \( x_k \) is the state of the \( k^{th} \) component and \( m \) is the total number of components. The set of all possible states \( x \), arising from combinations of all possible component states is denoted by \( X \), the state space.

The failure / success status of a given state can be obtained by calculating the maximum power flow going from the source node \( S \) to the terminal node \( T \), taking into account the power balance at each node and the arc limits. If the maximum flow is equal to the total demand, this means that all demand arcs arriving at \( T \) are at their limits. Therefore, all area loads are being supplied, i.e., there is no load curtailment.

Conversely, if the maximum flow is smaller than the total demand, it means that at least one of the area loads is not fully supplied. The amount of load curtailment is the difference between the total demand and the maximum flow value.

An alternative way of solving this problem is to find the minimum capacity cut between the source and terminal nodes [3]. It can be shown that the maximum flow value is equal to the minimum capacity cut. A cut is a partition of the system areas into two disjoint subsets. The capacity of a cut is the sum of all component capacities that connect the two subsets.

One immediate consequence of this alternative method is that the set of arcs (generators or interconnections) in the minimum cut are the system “bottlenecks”. Thus, reinforcing the capacities of arcs that do not belong to the minimum cut will not relieve the load curtailment.

Another consequence is that all demand arcs in the minimum cut are at their limits, i.e., their area loads are met and can be called “safe”. These demand arcs correspond to all areas to the left of the minimum cut.

The areas to the right of the minimum cut are “unsafe”, i.e., subject to load curtailment. In other words, the minimum cut allows us to determine the area reliability indices identifying which areas are supplied and which are with deficits.

If cut I is the minimum cut, areas 1 and 2 will have load curtailments because of generation limits. In this case, the deficit of the system will not be reduced if any reinforcement in the interconnection is made. If cut IV is minimum both areas will have their load completely supplied.

If II is the minimum cut, area 1 is in the “unsafe” region, area 2 has its load supplied and the interconnection is a “bottleneck” to the overall deficit reduction. If III is the minimum cut the situation is reverse, and still the interconnection is a constraint to the system load supply. Note that in the first two cases (cuts I and IV) the interconnection does not lie in the minimum cuts.

**COPFT EVALUATION**

The capacity outage probability and frequency table (COPFT) of each generating arc \( j \) can be recursively evaluated through the following convolution expressions [1,2]:

\[
\begin{align*}
    p(x_j) &= p'((x_j - C_k) \times (1 - u_k) + p'((x_j) \times u_k) \quad \text{(1)} \\
    f^u_{x_j}(x_j) &= p'((x_j - C_k) \times f^u(C_k) + (1 - u_k) \times f^u((x_j - C_k))) \\
    &+ p'(x_j) \times f^u(0) + u_k \times f^{s'}(x_j) \quad \text{(2)}
\end{align*}
\]

where:

- \( p(x_j) \) and \( p'(x_j) \) are the probabilities of available generating capacity equal to \( x_j \), after and before the addition of \( k^{th} \) unit to the table;
- \( f^u_{x_j}(x_j) \) and \( f^{s'}_{x_j}(x_j) \) are the incremental frequencies of available generating capacity equal to \( x_j \), after and before the addition of \( k^{th} \) unit to the table;
- \( f^w(C_k) \) and \( f^w(0) \) are the incremental frequencies associated to the “up” (capacity equal to \( C_k \)) and “down” states of the units.

The incremental frequency associated to a state can also be expressed as the product between the probability and incremental transition rate of that state [4,5]. For example, the incremental frequency associated to an equipment \( e_k \) is given by:

\[
    f^{\text{in}}(e_k) = p(e_k) \cdot A^{\text{in}}(e_k)
\]

where:

Figure 1 - Multi-area representation of a two area system

Figure 2 shows a two-area system with all possible cuts.

Figure 2 - Possible cuts in a two area system
Expressions (3) and (4) are associated to transitions from failure to success states. We can also have an alternative formulation associated to transitions from success to failure states:

\[ \lambda_k^{in}(e_k) = \begin{cases} \mu_k & \text{if } e_k \text{ is down} \\ -\mu_k \times \frac{u_k}{1-u_k} & \text{if } e_k \text{ is up} \end{cases} \]  

(4)

Using (3) and (4), equation (2) is simplified to:

\[ f_j(x) = \left[ p'(x_j) - p'(x_j - C_i) \right] u_k \times \lambda_k + \left( 1-u_k \right) \times f_j^{in}(x_j) \]  

where

\[ f_j^{in}(x_j) = \left[ p'(x_j - C_i) - p'(x_j) \right] \left( 1-u_k \right) \times \lambda_k + \left( 1-u_k \right) \times f_j^{in}(x_j) \]  

(5)

Expressions (3) and (4) are associated to transitions from failure to success states. We can also have an alternative formulation associated to transitions from success to failure states:

\[ \lambda_k^{in}(e_k) = \begin{cases} -\lambda_k \times \frac{1-u_k}{u_k} & \text{if } e_k \text{ is down} \\ \lambda_k & \text{if } e_k \text{ is up} \end{cases} \]  

(6)

and

\[ f_j^{in}(x_j) = \left[ p'(x_j - C_i) - p'(x_j) \right] \left( 1-u_k \right) \times \lambda_k + \left( 1-u_k \right) \times f_j^{in}(x_j) \]  

(7)

### Evaluation of the Multi-Area Reliability Indexes

The problem of calculating reliability indices can be generally formulated as obtaining the expected value of a given test function [6]:

\[ E(F) = \sum_{x \in X} F(x) \times P(x) \]  

(8)

where \( P(x) \) is the probability of state \( x \) and \( F(x) \) is the test function; its objective is to verify whether that specific combination of area generation and interconnection capacities is able to supply one specific load. Since the system state vector is a random variable, the result is also a random variable.

### Test Functions for LOLP and EPNS Estimates

The LOLP index can be obtained as the expected value of the following indicator function:

\[ F(x) = I_f(x) = \begin{cases} 0 & \text{if } x \text{ is a success state} \\ 1 & \text{if } x \text{ is a failure state} \end{cases} \]  

(9)

The indicator function \( I_f(x) \) is based on the identification of the failure states. The expected power no supplied, EPNS, is calculated making \( F(x) \) equal to the required load curtailment associated to a given state \( x \).

### Test Function for LOLF Estimate

References [4,5] proposed a formulation for the LOLF test function, based on conditional probability and incremental frequency concepts [7,8,9]. This function does not require the explicit identification of the boundary crossings of each state. We can have different formulations for failure or success states.

#### Formulation 1 - system failure states

\[ F(x) = \lambda_f^{in}(x) = I_f(x) \times \sum_{k=1}^{n} \lambda_f^{in}(x_k) \]  

(10)

where \( \lambda_f^{in}(x_k) \) is the incremental transition rate [4] associated to transitions from failure to success states and \( \lambda_f^{in}(x_k) \) is defined as:

\[ \lambda_k^{in}(x_k) = \begin{cases} \mu_k & \text{if } x_k \text{ is down} \\ -\mu_k \times \frac{u_k}{1-u_k} & \text{if } x_k \text{ is up} \end{cases} \]  

(11)

#### Formulation 2 - system success states

\[ F(x) = \lambda_s^{in}(x) = I_s(x) \times \sum_{k=1}^{n} \lambda_s^{in}(x_k) \]  

(12)

where

\[ \lambda_k^{in}(x_k) = \begin{cases} -\lambda_k \times \frac{1-u_k}{u_k} & \text{if } x_k \text{ is down} \\ \lambda_k & \text{if } x_k \text{ is up} \end{cases} \]  

(13)

and the indicator function \( I_s(x) = 1 - I_f(x) \) is now based on the identification of the success states.

### Outline of Monte-Carlo Simulation

The reliability indices can also be estimated by a non-sequential Monte-Carlo simulation. In this approach, states \( x \in X \) are sampled from their joint probability distributions. The expected value of any test-function \( F(x) \), \( E(F) \), is estimated as:

\[ \tilde{E}(F) = (1 / NS) \sum_{i=1}^{NS} F(x_i) \]  

(14)

It is important to observe in (14), \( \tilde{E}(F) \) is not the “true” (population) expected value \( E(F) \), but an estimate of its value. In other words, if the experiment is repeated with a different random sample, a different value would be obtained for the estimate. Since \( F(x) \) and \( \mu(x) \) are random variables, \( E(F) \) is also a random variable. The uncertainty related to the estimation is given by the variance of the estimator:

\[ V(\tilde{E}(F)) = V(F) / NS \]  

(15)

where \( V(F) \) is the variance of the test-function.

Expression (15) indicates that uncertainty of the estimates depends on the variance of the test-function \( V(F) \) and is inversely proportional to the sample size. This confirms the intuitive notion that the accuracy of a Monte Carlo experiment increases with larger sample size \( NS \). This means that the the
sample size to estimate, for example, the LOLP of a two area system can be the same to evaluate the LOLP of a ten area system, if the same accuracy is required.

This uncertainty is often represented as a coefficient of variation:

$$\beta = \sqrt{V(\hat{E}(F)) / \hat{E}(F)}$$  \hspace{1cm} (16)

The coefficient of variation is used as a measure of the accuracy of the estimates.

Monte-Carlo simulation methods are very useful since we can obtain estimates of reliability indices, the associated variance and probability distributions of important variables (the expected power not supplied, for example).

**GENERATION RELIABILITY ANALYSIS IN THE BRAZILIAN SYSTEM**

The indexes presently used to evaluate the peak load supply in the interconnected systems are: loss of load probability (LOLP), expected power not supplied (EPNS), loss of load expectation (LOLE), loss of load frequency (LOLF) and loss of load duration (LOLD).

In addition to these well known indexes, there are interconnections sensitivity indexes. These indexes identify, among the main links, which of them can contribute to the reduction of the global reliability index in case of reinforcements.

The probability distribution function of the EPNS is aggregated to this set of indexes and gathers important information about the amount of possible deficits and the associated risk.

**Annual Operation Plan - AOP**

The Annual Operation Plan defines, as main results, the energy and peak contracts between utilities and the evaluation of the conditions of energy and peak load supply for the whole system [10].

In order to obtain the main predictive indexes for energy and peak supply, this step takes into account several factors like: different inflow scenarios (historic data for 64 years), maintenance schedule for all generating units, energy and peak load forecasts, interconnection limits between subsystems and areas, generation expansion program and the expected storage levels at the beginning of the planning horizon.

In the long-term operations planning, the two Brazilian systems are divided in major interconnected areas to represent bulk transmission links (Fig. 3 and 4). The CONFINT methodology [11] is applied, thus resulting in generation reliability indexes for each area and for the global system.

**Monthly Operation Program - MOP**

The main objectives of the short-term studies are: establish generation targets for each thermal plant and energy exchanges between subsystems; guidelines for the storage levels of each reservoir; turbine releases and spillage for each hydro plant; the refinement of the energy and peak long-term supplies and the possibility to offer secondary energy [12].

These studies indicate periods of time when it is possible to expect difficulties for the peak load supply. The alternatives for operation can be: management of maintenance schedules, demand management and even indication of the necessity of transmission reinforcements.

**CASE STUDY**

Since September 1996, this methodology has been applied to the South - Southeast system as a step of the Monthly Operation Program. The analysis has been made based on the following assumptions:

- the South - Southeast system was divided in two major areas with a 2800 MW limit capacity for the SE-S connection and 4800 MW limit capacity for the S-SE connection;
- maximum weekly peak load data obtained from the utilities;
- maintenance unit schedule available with the GTPR;
- power plants peak availability given by the daily operation personnel (most updated information);
- the duration of the peak load curve is 30 hours;
- statistical data for unavailability of generating units from the Brazilian Committee for Statistical Analysis (BRACIER).
Figure 5 presents the available generation resources in the Annual operation plan (maximum, medium and minimum for the historic inflow series) and in the Monthly operation program (MOP).

Figure 5 – Available generation resources (MW) during 1997

Figure 6 shows the peak load forecasts at the AOP stage and the monthly forecasts of the MOP. From April to the end of the year the forecasts have been quite close, with a maximum deviation around 4%.

Figure 6 – Peak load forecasts (MW) - 1997

Among the input parameters to evaluate the reliability analysis, the maintenance scheduled is one that planners have more influence to change. It can be seen that the figures of the AOP have been re-scheduled in order to reduce the risks of peak load deficits.

Figure 7 – Maintenance schedules (MW) - 1997

The difference in the input parameters from the Annual operation plan step and the Monthly operation programs has major influence in the reliability indexes, as it can be noticed in Figures 8 and 9.

It can be noticed that the maintenance reallocation during 1997 increased the resources capacity to supply the peak load, thus reducing the reliability indexes during this year.

Important information that is now available at each run of the CONFINT model are the probability density and distribution functions of the EPNS index. With these curves, planners have a more accurate figure of this index and what measures can be taken.

Figure 10 illustrates the probability distribution function of the EPNS and the probability density function of the deficits along the sampled events for the fifth week of June, 1997. Both curves start at 0 MW deficit and 0.9041 probability (1-LOLP is the probability of the occurrence of no deficits).

Figure 10 - Density and Distribution Functions of the EPNS of 5th week
Figure 11 illustrates the same probability density and the probability distribution functions conditioned to the occurrence of deficits. These curves are useful if one is interested in obtaining the distribution of deficits knowing that a deficit has occurred.

The analysis of these curves can help to indicate several levels of predictive load curtailments and the associated risks, supplying the personnel responsible for operations planning with these additional information.

**FUTURE IMPLEMENTATIONS**

A seven area approach will be implemented to Monthly Operation Program, in 1998, in order to apply the same representation used in long term studies.

Another implementation foreseen for the next year is the calculation of the exact sensitivities of multi-area reliability indexes with respect to variations of equipment failure and repair rates. These sensitivities allow ranking equipment in terms of their effect on system reliability indexes as well as estimating the impact of parameter uncertainty on reliability indexes.

**CONCLUSIONS**

This paper presents the application of a methodology with relevant information for the short-term operations planning process. Difficult periods for the peak load supply can be predicted with this methodology. The main point is the quantification of parameters like risks, deficits, frequency and probability functions.

Knowing these information and parameters has become more important, specially with the present changes in structure and management of the Brazilian electric system.

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